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INFALLIBILITY VERSUS UNIQUENESS IN DARE ANALYSES OF HAZARDOUS WASTE RISK

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Summary

Decision Alternative Ratio Evaluation (DARE) analysis is one technique for developing cardinal scale risk ratings of hazardous waste alternatives. Although no procedure can resolve all the difficulties of evaluating hazardous waste risk, DARE offers several distinct advantages including the elimination of unmanageable conservativism, the accomodation of information uncertainty, and the dimensional independence to integrate across diverse hazard types. In this paper, the DARE analysis of uncertain "raw" information is developed using probabilistic set membership. It is demonstrated that, although this introduces fallibility into the analysis, it provides a practical method for reducing the non-uniqueness observed in existing DARE procedures based on bounded fuzzy set membership.

1. Introduction

The analysis of hazardous waste risks poses extremely difficult problems. People (including experts) can seldom agree on the appropriate hazard set. Normally the candidates are numerous, and span the full range from obvious, acute, point hazards to distributed, chronic, nearly undetectable, and sometimes completely metaphysical dangers. In addition, people seldom agree on how the likelihood (probability) of hazard realizations should be quantified. From human toxicology to structural dynamics, the techniques of associating probabilities with hazards are thoroughly debatable. Most hazardous waste technologies are also quite new. We simply don't have a long history (successful or otherwise) from which to extract data. Finally, even if the individual risks of hazards can be quantified (risk $=f$(probability) $\times f$(severity)) there is seldom agreement on how to composite these values into a final ranking or rating.

It is also very difficult to coalesce the contrivances of all hazardous waste risks into one comparable set of units. Aesthetic and environmental degradation simply cannot be expressed in terms equivalent to human morbidity or mortality. This is very unlike cost analysis where dollars play a nondebatable,

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unifying role. Although people will argue about the cost or value of all elements of an economic analysis, they normally do not debate the procedure of summing dollars to evaluate the final result. This is not the case with risk. As William Lowrance (1976) has pointed out, people are extremely "human" when it comes to the mathematics of risk. What may be computationally equivalent risks will seldom be perceived of as equals. Lowrance (1976) went on to describe 10 factors that are essential to risk perception, and capable of defying conventional mathematics. Risks entered into voluntarily such as occupational or recreational exposures will be perceived as being less important than an equal amount of involuntary exposure. Unavoidable risks such as driving an automobile are often perceived of much differently than luxury exposures such as flying. Nearly all controversies involving hazardous wastes fall into the "dread hazard" category dominated by emotional over rational behavior. This has had a profound impact on the evolution of hazardous waste management in the 1980's.

All of this substantially compounds the evaluation of hazardous waste risks. Waste management technologies can be sophisticated, the potential dangers are numerous, the availability of incontrovertible data is limited, and the very thought of hazardous waste invokes strong personal emotion. The author is not aware of any solution for all of these quandaries, but Decision Alternative Ratio Evaluation (DARE) (Klee, 1971) offers compelling advantages over many existing techniques. Principal among these are the formalized structure imposed on the problem statement, the elimination of unmanageable conservativism, the incorporation of information uncertainty, the integration of nondimensional rankings across diverse hazard types, and the generation of results that may be meaningfully composited into final cardinal scale ratings.

Jennings and Sholar (1984), and Jennings and Suresh (1984), describe deterministic DARE-based techniques for developing risk penalty functions for hazardous waste planning. Jennings and Suresh (1986a) extended the technique to analytical solutions for the "fuzzy information" problem. By this procedure the extreme rating possibilities may be constructed about risk ratings that are fuzzy due to uncertain information known to fall within a bounded set of possibilities. Because of the substantial increase in computational complexity, the algorithms have been formulated for user-interactive microcomputer implementation (Jennings and Suresh, 1986b).

The purpose of this paper is to extend DARE analysis into the regime of "fallible" solutions. It will be illustrated that, although the ratings computed from bounded-set fuzziness by the algorithms of Jennings and Suresh (1986a) are infallible, they can also be quite conservative. If one is willing to introduce a small probability of error, several alternatives for expressing information uncertainty may be introduced and the bounds about implied fuzzy risk ratings can be substantially reduced. Intuitively it may seem that the introduction of error is not an improvement. In reality, any hazardous waste risk analysis is at its very best only an approximation. It will be illustrated that the introduction and control of the possibility of error can be used to resolve problems of apparent non-uniqueness and distinguish these from truly non-unique fuzzy DARE ratings.

2. DARE analysis for bounded set fuzziness

The mathematics of DARE analysis for deterministic information are relatively straightforward. This is not to imply that their implementation is simple. Everything about hazardous waste problems is complex and debatable. Identifying a tractable set of "raw" information can be extremely difficult. However, once the information demands of DARE analysis have been satisfied, the actual solution procedure adds little additional burden.

DARE risk analysis may be applied to any set of decision variables (e.g. alternative site locations or alternative management technologies) considering any desired set of weighted decision criteria (depth to water table, toxicity of wastes, proximity of populations, etc.). Here "raw" information implies the definition of decision variables and criteria as well as the data necessary to compare them. This may be any desired combination of primary information such as laboratory or field measurements and/or the results of existing specialized methods for comparing hazardous waste activities (see Booz–Allen Applied Research, 1973; Gabor and Griffith, 1980; Hallstedt et al., 1986; Harris et al., 1984; Jones, 1977/78; Klee, 1976; Pack et al., 1987; Pavoni et al., 1972; Petts et al., 1987; Wu and Hilger, 1984; etc.). More mechanistic, problem-specific models can also be applied to yield "raw" DARE information. This high degree of flexibility results from the nondimensional cardinal ratings computed for each decision criterion.

The DARE analysis procedure may be generalized as follows:

- Let I be the number of decision variables and J be the number of decision criteria to be applied to these variables $(I > 1, J \geqslant 1)$.
- Let W_j be a nondimensional weight assigned to each decision criterion.

$$0.0 < W_j \leqslant 1.0 \qquad \forall j = 1, J \tag{1}$$

$$\sum W_j = 1.0 \tag{2}$$

- Let U_{ij} be the magnitude ratio of the state variable (ξ_j) quantifying the j^{th} decision criterion for the i^{th} decision variable relative to the $(i+1)^{\text{th}}$ variable.

$$U_{ij} = \frac{\xi_j\,(\text{decision variable } i)}{\xi_j\,(\text{decision variable } i+1)} \ \forall i = 1, (I-1); j = 1, J \tag{3}$$

$$0.0 < U_{ij} \ll \infty \tag{4}$$

$$U_{Ij} = 1.0 \tag{5}$$

Equation (5) fills the final row of the matrix $\boldsymbol{U}$ with an element passive to hierarchy construction and is required because the hierarchy for each criterion is uniquely defined by $(I-1)$ ratio evaluations. The constraint of eqn. (4) states that no alternative may be infinitely more or less preferable than any other alternative. From this "raw" information, the implied hierarchy may be constructed and normalized as follows.

$$H_{ij} = \pi_{i}\, U_{ij} \qquad \forall i=1,I; j=1,J \tag{6}$$

$$F_{ij} = H_{ij} / \sum_{i} H_{ij} \qquad \forall i=1,I; j=1,J \tag{7}$$

The final cardinal scale rating vector $\boldsymbol{R}$ may then be computed as

$$R_i = \sum_{j} F_{ij} W_j \qquad \forall i=1,I \tag{8}$$

or simply,

$$\boldsymbol{R}=\boldsymbol{F}\boldsymbol{W} \tag{9}$$

The computations are more complex when the raw information is fuzzy. Fuzziness implies that for some reason (precision of measurements, paucity of data, incomplete understanding of phenomena, honest skepticism, etc.) it is not possible to specify deterministic values for decision variable attribute ratio comparisons. However, given that one can place finite bounds on the magnitude of the required values, (subject to the constraints of eqns. (2), (4) and (5))

$$W_j^- \leqslant \tilde{W}_j \leqslant W_j^+ \qquad \forall j=1,J \tag{10}$$

$$U_{ij}^- \leqslant \tilde{U}_{ij} \leqslant U_{ij}^+ \qquad \forall i=1,(I-1)\ ;\ \forall j=1,J \tag{11}$$

infallible bounds about the implied ratings may still be computed.

For the i^{th} decision variable, the composition of the fuzzy evaluation matrix ($\tilde{\boldsymbol{U}}$) required to extremize its ultimate rating may be assembled from prescribed combinations of the extreme element values.

$$\tilde{U}_{ij}(\max)^i \begin{Bmatrix} U_{kj}^+ & \forall k=i,I; j=1,J \\ U_{kj}^- & \forall k=1,(I-1); j=1,J \end{Bmatrix} \qquad \forall i=1,I \tag{12}$$

$$\tilde{U}_{ij}(\min)^i \begin{Bmatrix} U_{kj}^- & \forall k=i,I; j=1,J \\ U_{kj}^+ & \forall k=1,(I-1); j=1,J \end{Bmatrix} \qquad \forall i=1,I \tag{13}$$

For the special case of fuzzy evaluations subject to deterministic weights, the implied ranking bounds may be computed by eqns. (14) and (15).

$$R_i(\max) = [\tilde{\boldsymbol{F}}_i(\max)^i]^{\mathrm{T}}\boldsymbol{W} \qquad \forall i=1,I \tag{14}$$

$$R_i(\min) = [\tilde{\boldsymbol{F}}_i(\min)^i]^{\mathrm{T}}\boldsymbol{W} \qquad \forall i=1,I \tag{15}$$

The vectors $\tilde{\boldsymbol{F}}_i(\max)^i$ and $\tilde{\boldsymbol{F}}_i(\min)^i$ are the i^{th} row of the fuzzy hierarchy matrices $\tilde{\boldsymbol{F}}(\min)^i$ and $\tilde{\boldsymbol{F}}(\max)^i$ computed by eqns. (6) and (7) using the $\boldsymbol{U}(\max)^i$ and $\tilde{\boldsymbol{U}}(\min)^i$ matrices respectively. All these computations must be repeated for each decision variable.

DARE computations become more complex when weights are also fuzzy. Algorithms sufficient to unravel the fuzzy weighting problem, are presented in Jennings and Suresh (1986a) and will not be reiterated here. Let it suffice to say that infallible ranking bounds may still be computed if the correct adjustment is made to weighting vector, $\tilde{\boldsymbol{W}}$.

$$R_i(\max) = [\tilde{\boldsymbol{F}}_i(\max)^i]^{\mathrm{T}}(\boldsymbol{W}(\min) + \boldsymbol{W}(\max)^i) \qquad \forall \mathrm{i}=1,I \tag{16}$$

$$\mathrm{R}_i(\min) = [\tilde{\boldsymbol{F}}_i(\min)^i]^{\mathrm{T}}(\boldsymbol{W}(\min) + \boldsymbol{W}(\min)^i) \qquad \forall \mathrm{i}=1,I \tag{17}$$

Again, all calculations must be repeated for each of decision variables. The number and complexity of the calculations argue convincingly for a computer implementation. Jennings and Suresh (1986b) document microcomputer codes designed for this purpose.

3. Example "infallible" solution for bounded set fuzziness

For the purpose of illustration and comparison, the following "infallible" DARE solution is offered. The results are infallible in the sense that if the raw information is correct, the implied fuzzy ratings must fall within their computed bounds. The problem is presented in generic form since only the magnitude of the numbers are significant to this discussion. This does, however, correspond to the example of Jennings and Suresh (1986b) for eight hazardous waste technologies evaluated for five general classes of hazard.

Assume that 5 decision criteria ($j=1,5$) have been identified and assigned the fuzzy weights indicated in Table 1. Also assume that 8 decision variables ($i=1,8$) have been evaluated to yield the pairwise $\tilde{U}_{ij}$ bounds of Table 2.

Bounded-set DARE yields the infallible rating bounds presented in Table 3. These bounds are illustrated graphically in Fig. 1. Note that although one could

TABLE 1

Example fuzzy decision criteria weight bound assignments

Criteria (j)	W_j^-	Mean	W_j^+	Deviation
1	0.0900	0.10	0.1100	($\pm$10%)
2	0.1125	0.15	0.1875	($\pm$25%)
3	0.1800	0.30	0.4200	($\pm$40%)
4	0.1125	0.15	0.1875	($\pm$25%)
5	0.1800	0.30	0.4200	($\pm$40%)

TABLE 2

Example fuzzy decision alternative ratio evaluation information

Variables	Ratio evaluation bounds for decision criteria $[U_{ij}^- - U_{ij}^+]$				
	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
$i=1$	1.350–1.650	1.275–1.725	0.180– 0.420	0.170– 0.230	0.060– 0.140
$i=2$	2.250–2.750	1.700–2.300	0.600– 1.400	1.700– 2.300	1.200– 2.800
$i=3$	0.180–0.220	0.170–0.230	0.600– 1.400	0.425– 0.575	0.300– 0.700
$i=4$	1.800–2.200	4.250–5.750	6.000–14.00	8.500–11.50	6.000–14.00
$i=5$	0.450–0.550	0.425–0.575	0.150– 0.350	0.425– 0.575	0.150– 0.350
$i=6$	0.900–1.100	1.700–2.300	0.300– 0.700	1.700– 2.300	0.300– 0.700
$i=7$	0.900–1.100	4.250–5.750	1.200– 2.800	4.250– 5.750	1.200– 2.800
$i=8$	1.0	1.0	1.0	1.0	1.0

draw general conclusions about risk for these alternatives, the overlapping rating domains make it very difficult to identify even a distinct ordinal preference hierarchy. To emphasize this point, the non-uniqueness of ratings in the range of 0.116 to 0.161 has been highlighted on Fig. 1. Note that ratings for 7 of the 8 decision variables could fall within this range and are therefore indistinguishable from one another.

4. Probabilistic set membership functions for fuzzy information

The rating extremes of Table 3 and Fig. 1 are infallible in the sense that, given the raw data, it is not mathematically possible for ratings to fall beyond the computed bounds ($R_i(\min) \leqslant \tilde{R}_i \leqslant R_i(\max)$; $\forall i=1,I$). The advantages of being able to conduct risk analysis while being honest about information quality are obvious. The resulting ratings can also be inconvenient (as in this example) if they suffer from strong non-uniqueness. This cannot always be avoided. Figure 1 also presents the deterministic ratings implied if all uncertain data are assigned a deterministic value midway between their set bounds. Although some of the distinctions between alternatives become more obvious (clearly #4 receives the highest rating and #5 receives the lowest rating) the differences between #3 and #7 or #1 and #8 remain practically indistinguishable.

It is always possible that a fuzzy rating ($\tilde{R}_i$) should be assigned one of its theoretical extremes. It is also intuitively obvious that this possibility is very unlikely. For this to occur for any variable of the example problem, each of 40 fuzzy coefficients must take on a specific extreme value. Furthermore, the occurrence of an extreme rating ($R_i(\min)$ or $R_i(\max)$) for one variable precludes the occurrence of this same extreme for any other variables.

To quantify the probability of a decision variable taking on one of its theoretical extremes it is necessary to depart from the original fuzzy set concept of

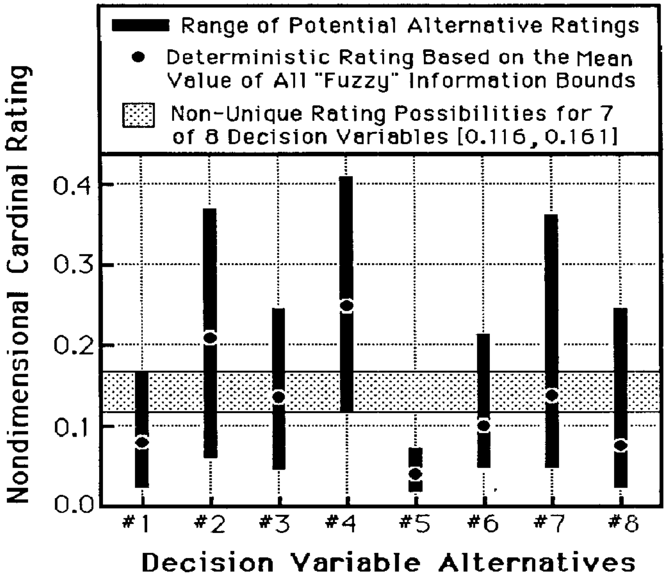

Fig. 1. Example "fuzzy" decision variable rating exhibiting strong non-uniqueness.

TABLE 3

Rating bounds implied by the example fuzzy DARE analysis problem

Decision variable	Rating bounds	Decision variable	Rating bounds
$i=1$	0.022-0.161	$i=5$	0.019-0.067
$i=2$	0.059-0.366	$i=6$	0.046-0.209
$i=3$	0.045-0.240	$i=7$	0.048-0.358
$i=4$	0.116-0.405	$i=8$	0.023-0.241

Zadeh (1965) and associate probability (P) with set membership. One very simple approach for this is to assume that fuzzy variables are discrete and must take one of their set bounds.

$$P(W_j^-)+P(W_j^+)=1.0 \qquad \forall j=1,J \tag{18}$$

$$P(U_{ij}^-)+P(U_{ij}^+)=1.0 \qquad \forall i=1,(I-1)\ ;\ j=1,J \tag{19}$$

If it is further assumed that all probabilities are equal, the probability of any rating ($\tilde{R}_j$) taking on one of its theoretical extremes is given by eqn. (20). Clearly although this is possible, it is also negligible as a practical consideration.

$$P[(\tilde{R}_i=R_i(\min))U(\tilde{R}_i=R_i(\max))]=2(0.50)^{40}<2.0\times10^{-12} \tag{20}$$

The discrete binary probabilistic membership model of eqns. (18) and (19) is simple, but plausible for some applications. Consider that two different analytical models or two independent field measurements might be available to develop a ratio comparison U_{ij}, and this might be the only information available. It would not be unreasonable to assume that one of the values is correct. It might even be illogical or mathematically incorrect to consider intermediate values (such as the average of the two integer values) if they do not or could not exist. However, without knowledge of which datum is correct, one would have to assign some probability of correctness to each. This may also be generalized to any number of discrete intermediate values (see Fig. 2).

It is often more realistic to consider any "known" value as the sample of a continuous random variable. When $\tilde{W}_j$ and $\tilde{U}_{ij}$ are assumed to be continuous, the alternatives for quantifying set membership expand considerably. Example continuous membership models appropriate for DARE analysis are illustrated in Fig. 2.

The example computations presented are based on the uniform Probability Density Function (PDF) membership concept of Fig. 2. Given no additional information except that the variable is random and bounded, the uniform probability density is the simplest assumption. It is also easy to visualize situations where symmetric, nonlinear PDF's (Gaussian, Student's t, Laplace, etc.) would be more appropriate to quantify randomness. These often apply when non-systematic randomness is imparted by imperfect measurement. It is also quite plausible that skewed PDF's (i.e. beta, gamma, Johnson's, chi square, Gumbel, Weibull, Rayleigh, F, exponential, etc.) could apply in more subjective applications or where systematic errors skew results. The reader is

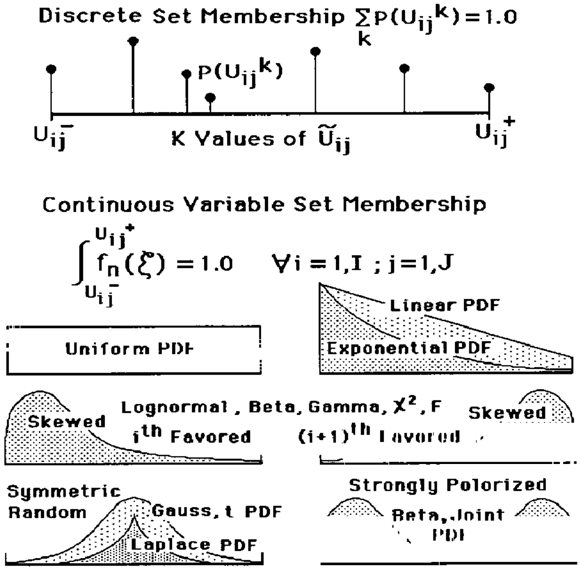

Fig. 2. Schematic illustration of PDF alternatives for probabilistic set membership.

directed to the excellent works of Johnson and Kots (1970) for a thorough presentation of the classical mathematics of PDF's.

5. Example "fallible" solutions for bounded set fuzziness

To illustrate the potential of probabilistic DARE approaches to hazardous waste risk analysis, let the example problem statement be modified by specifying a uniform probability density across all fuzzy data sets. This requires no new "raw" information, but implies that the fuzzy coefficients ($\tilde{W}_j$, $\tilde{U}_{ij}$) are continuous, uniformly distributed random variables bounded by the upper and lower limits of Tables 1 and 2. Unfortunately, although the problem extension into probabilistic set membership is easily accomplished, its solution is not.

For the purpose of this discussion, numerical solutions were generated by "brute force" Monte Carlo simulation (see Kalos and Whitlock, 1986; Sobol, 1974). Brute-force simulation implies that no problem specific information or dynamic statistical analyses were applied to accelerate solution convergence. For each Monte Carlo realization, random weights ($\tilde{W}_j$) and ratio evaluations ($\tilde{U}_{ij}$) were sampled by scaling randomly generated uniform variates. Uniform variates were generated using the pseudo-random linear congruential generator technique of Lehmuer (see Hoaglin, 1983) with coefficients as recommended by Gordon (1978). This will be referred to as a "fallible" solution of the rating problem since there is always some possibility (albeit small) of the ratings falling beyond the computed results.

Figure 3 presents histograms of $\tilde{R}_i$ computed from 10,000 Monte Carlo realizations of the example problem. Histograms were quantified as dynamic summations by dividing the infallible rating domain ($R_i(\max)-R_i(\min)$; $\forall i=1,I$) into 50 discrete intervals. Although this fails to maximize resolution, it does avoid the necessity of saving the huge volume of realization data, and yields accuracy sufficient for most practical applications. The now vertical, non-unique rating regime of Fig. 1 [$0.116 \leqslant \tilde{R}_i \leqslant 0.161$] has also been emphasized. From Fig. 3 it is clear that, although it is theoretically possible for any variable except #5 to yield a rating in the non-unique regime, it is very improbable ($P \leqslant 0.06$) that the rating for any alternative except #3 or #7 actually belongs in this range.

Table 4 provides a comparison of the infallible rating bounds and the bounds required to contain the 10,000 fallible realizations of Fig. 3. Note that for all decision variables the bounds may be reduced by over 50 percent. Although this data is not sufficient to determine the probability of error (i.e. of a rating falling beyond a fallible bound), it must be a small number somewhere in the neighborhood of 1/10,000.

The results of Table 4 clearly demonstrate the major point of this paper. The infallible bounds computed by the algorithm of Jennings and Suresh (1986a)

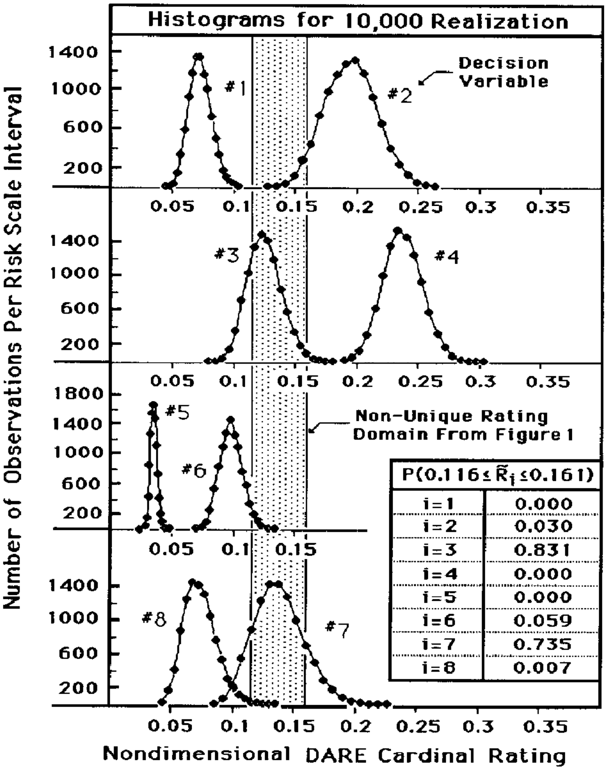

Fig. 3. Histogram approximations of probabilistic DARE ratings.

TABLE 4

Comparison between infallible and fallible rating bounds

Alternative	Infallible bounds	Fallible bounds*	Range (%)**
$i=1$	0.022–0.161	0.048–0.106	41.7
$i=2$	0.059–0.366	0.134–0.264	42.3
$i=3$	0.045–0.240	0.088–0.176	45.1
$i=4$	0.116–0.405	0.190–0.300	38.1
$i=5$	0.019–0.067	0.028–0.045	35.4
$i=6$	0.046–0.209	0.068–0.138	42.9
$i=7$	0.048–0.358	0.087–0.232	46.8
$i=8$	0.023–0.241	0.044–0.140	44.0

*Bounds containing $\tilde{R}_i$ for 10,000 Monte Carlo realizations.
*Percent of infallible range occupied by the fallible range.

yield very conservative answers. If one is willing to tolerate a small probability of error, these bounds can be substantially reduced.

The Monte Carlo solution technique applied here also implies additional

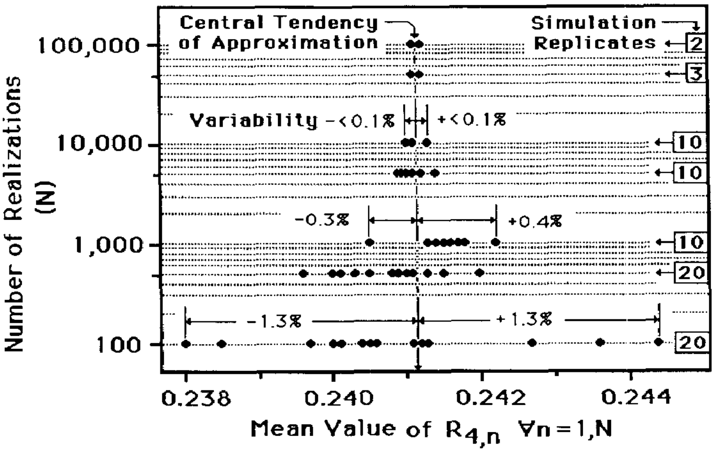

Fig. 4. Convergence to the mean rating of $\tilde{R}_4$ as a function of Monte Carlo realizations.

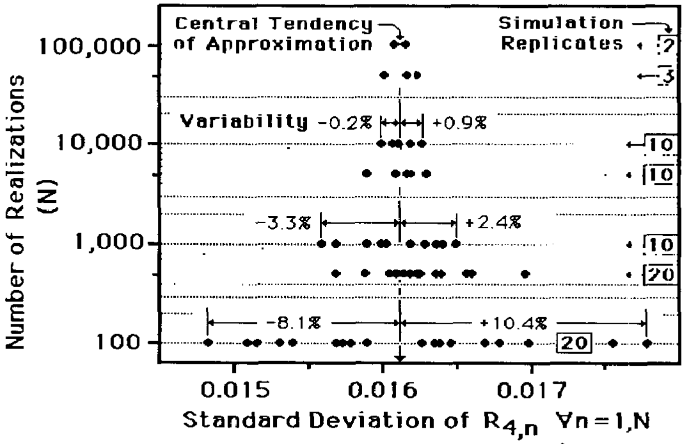

Fig. 5. Convergence to the standard deviation of $\tilde{R}_4$ as a function of Monte Carlo realizations.

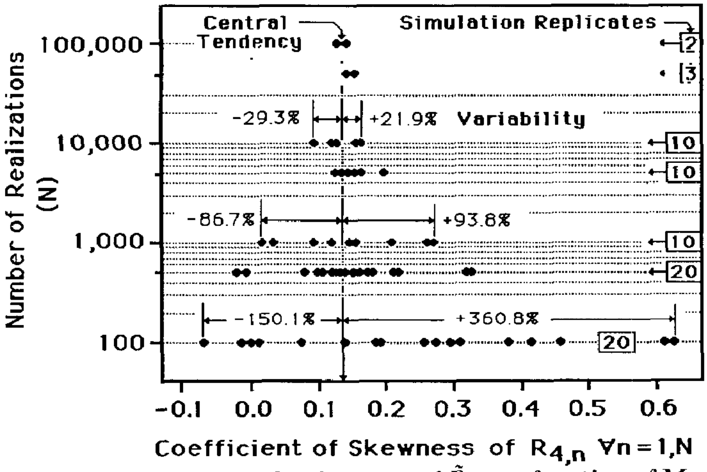

Fig. 6. Convergence to the skewness of $\tilde{R}_4$ as a function of Monte Carlo realizations.

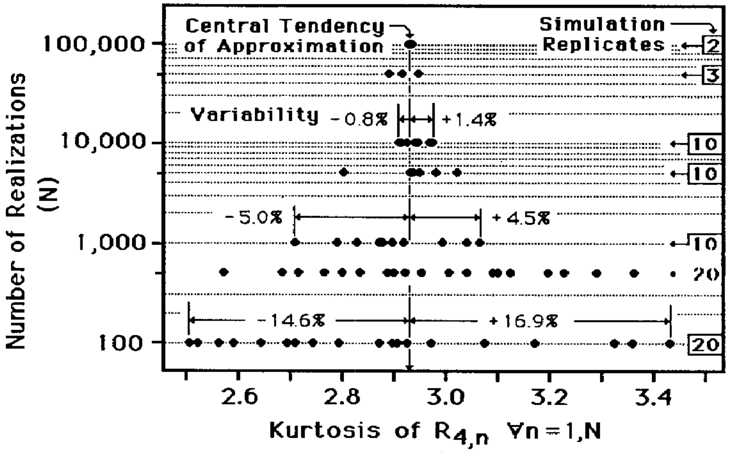

Fig. 7. Convergence to the kurtosis of $\tilde{R}_4$ as a function of Monte Carlo realizations.

TABLE 5

Statistics* based on the first four moments of $\tilde{R}_i$

Alternative	Mean (μ)	Deviation (σ)	Skewness (β_1)	Kurtosis (β_2)
$i=1$	0.075	0.009	0.304	2.99
$i=2$	0.199	0.021	0.109	2.78
$i=3$	0.128	0.014	0.260	2.99
$i=4$	0.241	0.016	0.136	2.94
$i=5$	0.036	0.003	0:230	2.96
$i=6$	0.100	0.009	0.296	3.03
$i=7$	0.143	0.020	0.370	3.03
$i=8$	0.078	0.013	0.574	3.36

*Computed from 100,000 Monte Carlo realizations.

questions and potential difficulties. The motivation for the algorithms of Jennings and Suresh (1986a) was to provide fast solutions to support microcomputer-based, user-interactive analysis. Solutions requiring large numbers of realizations hardly satisfy this criterion. The 10,000 realizations of Fig. 3 required approximately 20 minutes of PC-AT computer time using a reasonable efficient implementation program. Solutions based on 100,000 realizations required in excess of 3 hours. Computation times would be much larger for more extensive problems. Times of this magnitude often preclude microcomputer use, and would certainly eliminate user-interactive activities.

To investigate the sensitivity of Monte Carlo solutions to the number of realizations, and to explore possibilities for solution alternatives, the first four moments (M_i^w, $\forall w=1,4$; $i=1,I$) were computed for all risk realizations. Four accurate moments are sufficient to distinguish between most common, but not all continuous PDF's (Pearson, 1963). These moments may be computed from

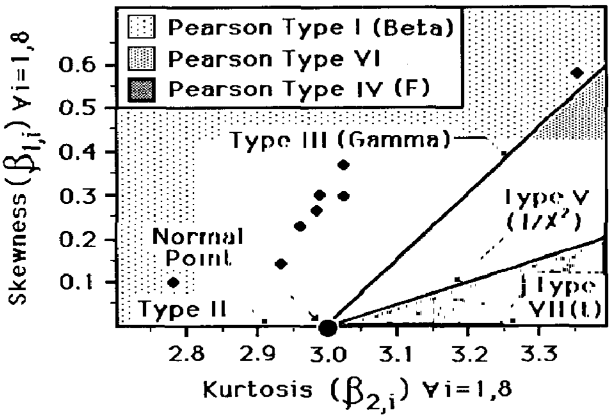

Fig. 8. Example results relative to the PDF solution regimes of Pearson's differential equation (Pearson and Hartley, 1972).

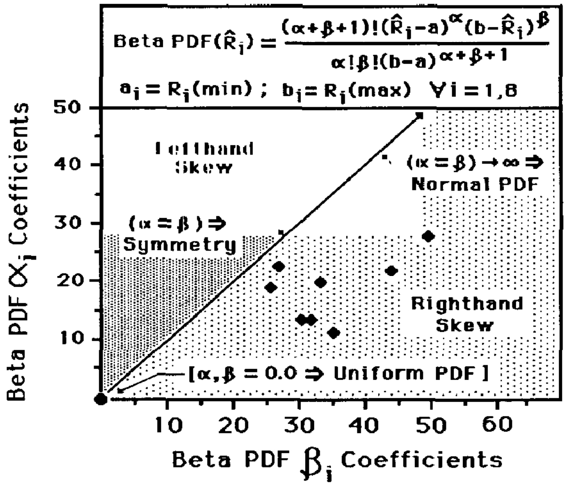

Fig. 9. Results of Beta function calibration (Harr, 1987).

the running totals (Σ on $n=1,N$ realizations) implied in the right-hand-most equalities of eqns. (21)–(24). Here $R_{i,n}$ represents the n^{th} realization of the random variable $\tilde{R}_i$. Note that eqns. (21)–(24) allow moments to be evaluated without storing realization data.

$$M_i^1 = \Sigma R_{i,n}/N \ ; \quad \forall i=1,I \tag{21}$$

$$M_i^2 = \sigma_i^2 = \Sigma(R_{i,n}-\mu_i)^2/N = \Sigma(R_{i,n})^2/N - \mu_i^2; \quad \forall i=1,I \tag{22}$$

$$M_i^3 = \Sigma(R_{i,n}-\mu_i)^3/N = \Sigma(R_{i,n})^3/N - 3\mu_i\Sigma(R_{i,n})^2/N + 2\mu_i^3 \ ; \quad \forall i=1,I \tag{23}$$

$$M_i^4 = \Sigma(R_{i,n} - \mu_i)^4/N$$
$$= \Sigma(R_{i,n})^4/N - 4\mu_i\Sigma(R_{i,n})^3/N + 6\mu_i^2\Sigma(R_{i,n})^2/N - 3\mu_i^4 \quad (24)$$

Mean: $\mu_i = M_i^1 \quad \forall i=1,I$ (25)

Standard deviation: $\sigma_i = (M_i^2)^{0.5} \quad \forall i=1,I$ (26)

Skewness: $\beta_{1,i} = M_i^3/(M_i^2)^{3/2} \quad \forall i=1,I$ (27)

Kurtosis: $\beta_{2,i} = M_i^4/(M_i^2)^2 \quad \forall i=1,I$ (28)

Figures 4 through 7 present data on convergence to the mean, standard deviation, skewness and kurtosis of the rating of alternative #4 as a function of the number of realizations attempted. Each point on these figures corresponds to an individual simulation of $100 \leqslant N \leqslant 100{,}000$ realizations. Replicates at each N indicate accuracy. Results as $N \to \infty$ indicate convergence. Note that accuracy deteriorates in the direction of higher moments and at different rates for odd and even orders. Many more realizations would be required if answers depend upon accurate propagation of higher moments. Table 5 presents values for the first four moments computed from 100,000 realizations. Observe that, although it is not evident in the histograms of Fig. 3, all results fail to satisfy the Gaussian distribution (skewness 0.0, kurtosis 3.0) and are skewed to the right (skewness > 0).

The skewness and kurtosis can be helpful in inferring a random variable's parent density function. Figure 8 illustrates the data of Table 5 plotted as a "Moment-Ratio" diagram in the neighborhood of the normal point (3.0) (see Johnson et al., 1963; McCuen, 1985; Pearson, 1963; Pearson and Hartley, 1972). The uniform distribution that defined all raw data is also a point in this space at ($\beta_1 = 0.0$, $\beta_2 = 1.8$) but does not appear on Fig. 8 because of the scales selected. All results for the eight decision variables plot in the domain of a Pearson Type 1 distribution which subsumes the Beta distribution (Johnson and Kotz, 1970). This is significant because, given deterministic bounds $[a,b]$, the Beta PDF may be calibrated using only the first two moments (μ,σ^2).

$$\text{Beta PDF: } f(\xi) = \frac{(\alpha+\beta+1)!(\xi-a)^{\alpha}(b-\xi)^{\beta}}{\alpha!\beta!(b-a)^{\alpha+\beta+1}} \quad (29)$$

$$\alpha(\mu,\sigma) = \frac{[(\mu-a)/(b-a)]^2\{1-[(\mu-\alpha)/(b-a)]\}}{[\sigma/(b-a)]^2} - \{1 - [(\mu-a)/(b-a)]\} \quad (30)$$

$$\beta(\mu,\sigma) = \frac{(\alpha+1)(b-a)}{\mu-a} - (\alpha+2) \quad (31)$$

It is very interesting to speculate that if probabilistic DARE analysis based on uniform PDF set membership always produced Beta distributed answers, then answer density functions could be calibrated using far fewer realizations. This could greatly accelerate the speed of microcomputer implementations. Figure 9 illustrates the results of a Beta PDF calibration for the example problem. As one should expect, all points plot in the regime of right hand skew (Harr, 1987), and intriguingly close to the uniform distribution point. Once the essential coefficients $(\alpha_i,\beta_i,a_i,b_i)$ have been computed with sufficient accuracy, all ranking probabilities may be computed directly from the integration of eqn. (29) without further numerical simulation.

6. Summary and conclusion

DARE risk analysis has proven to be a very useful tool for overcoming many of the quandaries of hazardous waste risk analysis. Although the technique yields relative rather than absolute values on a nondimensional risk scale, the results are sufficient to resolve many of the essential questions of hazardous waste management planning.

One of the more compelling difficulties of hazardous waste risk analysis is the management of uncertain information. Although there has been an intense concentration on nearly all aspects of hazardous wastes in the 1980's, knowledge has evolved (i.e. changed) so rapidly that it is difficult to distinguish fact from fallacy. Because of this, it is imperative that risk analyses acknowledge and accommodate uncertainty in the "facts" upon which they are based.

Jennings and Suresh (1986a) have presented microcomputer-based algorithms for accomplishing DARE risk analysis in the face of information uncertainty. Their bounded fuzzy set approach is capable of yielding infallible bounds around the implied risk ratings. Unfortunately, when the degree of information uncertainty is high, bounded set analysis can suffer from strong non-uniqueness resulting from the very conservative nature of infallibility. The procedures presented here attempt to extract more information about the practicalities of implied ratings by introducing a small probability of error. It has been illustrated that by adopting a model for probabilistic set membership, the implied risk ratings can be substantially refined. Although the introduction of potential error may seem to be a poor idea, risk analyses are, at their very best, only approximations. Even if all of the information used is precise, there is no guarantee that it is complete. Mathematic infallibility only yields the illusion of being correct.

The solution presented here does not resolve all remaining difficulties of probabilistic risk analysis. First, the user must be able to associate a density function with set membership. As illustrated in Fig. 3, there are many alternatives for this and one may not know *a priori* which is correct. Also, once probabilistic set membership has been defined, the analyst is faced with a substantially more difficult mathematical problem than that of bounded set analysis. The example solution presented here was based on homogeneous uniform distributions. Far more complex combinations of membership treatment are possible. Finally, although solutions may be generated by Monte Carlo simulations, the computational effort required for complex problems is of concern. More sophisticated solution implementation strategies will be required before probabilistic DARE analysis can be conveniently implemented.

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